General Physics: Electromagnetism, Problem Set 13

Exercise 1 :

At $t = 0$, a 12.0 V battery is connected in series with a 220 mH inductor and a total of 30 Ω resistance, as shown in the figure below.

1. What is the current at $t = 0$?
2. What is the time constant?
3. What is the maximum current?
4. How long will it take the current to reach half its maximum possible value?
5. At this instant, at what rate is energy being delivered by the battery and
6. At what rate is energy being stored in the inductor's magnetic field?

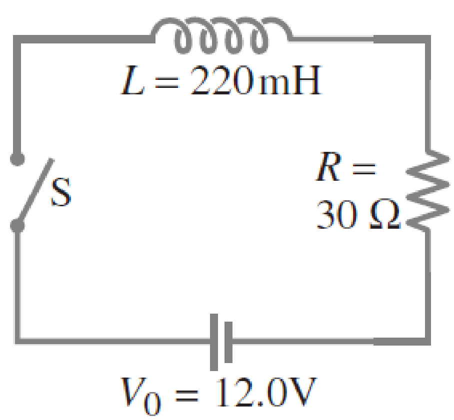

Figure 1: RL circuit.

Exercise 2 :

In the circuit below, the switch S is closed at $t = 0$.

1. Write down the differential equations for the currents in the circuit and find the equivalent RL circuit;

   Hint: Inductors in parallel combine the same way as resistors in parallel.

2. Compute the expression of the currents $i_1(t)$ and $i_2(t)$ flowing in the two inductances at a generic time t.

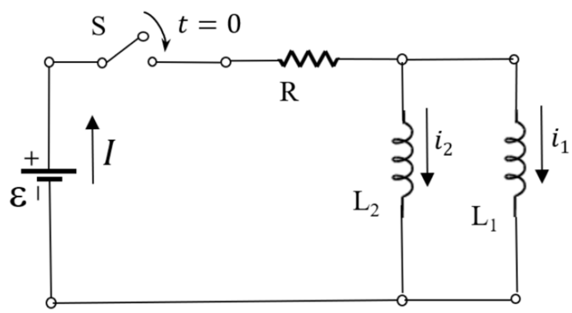

Figure 2: RL circuit.

Exercise 3 :

A 1200 pF capacitor is fully charged by a 500 V dc power supply. It is disconnected from the power supply and is connected, at $t = 0$, to a 75 mH inductor. Determine:

1. The initial charge on the capacitor;
2. The maximum current;
3. The frequency f and period T of oscillation;
4. The total energy oscillating in the system.

Exercise 4 :

The switch in the circuit shown in the figure below is held in position [a] for 2 seconds. The switch is then instantly moved to position [b] (without interrupting the electric current through the coil). At time $t = 2$ s, the capacitor carries no charge.

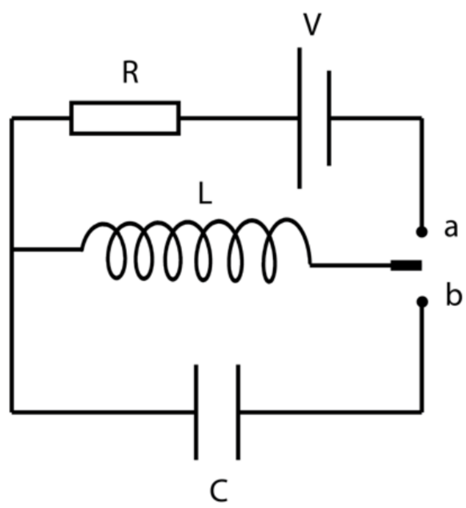

Figure 3: RL circuit.

In position [b] of the switch:

1. Calculate the electric current through the coil at time $t = 2$s.
2. Calculate the frequency of oscillation.
3. Determine the charge $Q(t)$ on the capacitor C as a function of time t.
4. Determine the current $I(t)$ as a function of time t.

Numerical application : $V = 0.2$ V, $R = 0.03\ \Omega$, $L = 54$ mH, $C = 3.2\ \mu$F.

Exercise 5 :

A conducting bar moves at a constant velocity v along two straight rails that form an angle α, in the presence of a uniform magnetic field B perpendicular to the triangle formed by the bar and the rails. The resistance of the bar is negligible, while the rails have a cross-sectional area S and are made of a material with resistivity ρ.

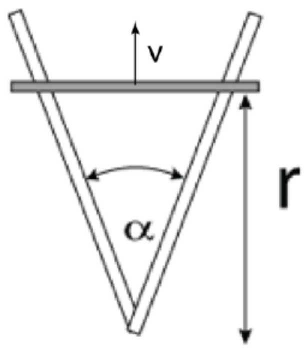

Figure 4: RL circuit.

1. Determine the amplitude of the current I induced in the bar when the distance between the bar and the vertex is r.
2. Determine the magnetic force on the moving bar as a function of time.

Initial conditions: $t = 0$ s and $r = 0$ cm. Numerical application $r = 12$ cm, $S = 1$ mm^2, $B = 0.2$ T, $\alpha = 50°$, $\rho = 1.5 \cdot 10^{-6}\ \Omega \cdot m$, $v = 30$ cm/s and $t = 0.4$ s.

Exercise 6 :

A 30 pF air-gap capacitor has two circular plates of area $A = 100\,\text{cm}^2$. It is charged by a 70 V battery through a $2.0\,\Omega$ resistor. In the instant the battery is connected, the electric field between the plates is changing most rapidly. In this instant, calculate:

1. The current into the plates;
2. The rate of change of electric field between the plates;
3. Determine the magnetic field induced between the plates. Assume $\vec{E}$ is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

Exercise 7 :

An induction stove contains a coil of copper wire underneath the ceramic plate, the "burner" (a burner that never gets hot). When a cooking pot is placed on top of it, an alternating perpendicular magnetic field is applied. The resulting oscillating magnetic field induce a magnetic flux changing in time, producing Eddy currents in the pot. These currents will heat up the pot due to Joule heating.
The heating power of an induction stove, designed for Switzerland, is $2\ kW$. What is the power of this stove if it is used in the USA with the same metal pan?
Hint:

- - CH: $V_{\text{eff,CH}} = 240$ V, $\omega_{\text{CH}} = 50$ Hz;
  - USA: $V_{\text{eff,USA}} = 115$ V, $\omega_{\text{USA}} = 60$ Hz;
- It can be useful to define an effective volage as follow, $V_{\text{eff}} = \frac{1}{T}\int_t^{t+T}(V_0\sin(\omega t'))^2 dt' = \frac{V_0}{\sqrt{2}}$.